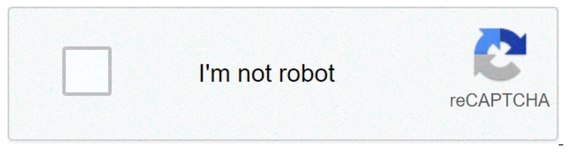

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How to find parallel lines cut by a transversal

Let’s explore why some angles are always equal. If I have two parallel lines cut by a transversal, I can identify alternate interior angles and use that to find missing angle measurements. Find the measure of angle . Explain or show your reasoning. Find and label a second degree angle in the diagram. Find and label an angle congruent to angle . Lines and are parallel. They are cut by transversal . With your partner, find the seven unknown angle measures in the diagram. Explain your reasoning. What do you notice about the angles with vertex and the angles with vertex ? Using what you noticed, find the measures of the four angles at point in the second diagram. Lines and are parallel. The next diagram resembles the first one, but the lines form slightly different angles. Work with your partner to find the six unknown angles with vertices at points and . What do you notice about the angles in this diagram as compared to the earlier diagram? How are the two diagrams different? How are they the same? Parallel lines and are cut by two transversals which intersect in the same point. Two angles are marked in the figure. Find the measure of the third angle. Lines and are parallel and is a transversal. Point is the midpoint of segment . Find a rigid transformation showing that angles and are congruent. In this picture, lines and are no longer parallel. is still the midpoint of segment . Does your argument in the earlier problem apply in this situation? Explain. When two lines intersect, vertical angles are equal and adjacent angles are supplementary, that is, their measures sum to 180 . For example, in this figure angles 1 and 3 are equal, angles 2 and 4 are equal, angles 1 and 4 are supplementary, and angles 2 and 3 are supplementary. When two parallel lines are cut by another line, called a transversal, two pairs of alternate interior angles are created. (“Interior” means on the inside, or between, the two parallel lines.) For example, in this figure angles 3 and 5 are alternate interior angles and angles 4 and 6 are also alternate interior angles. Alternate interior angles are equal because a rotation around the midpoint of the segment that joins their vertices takes each angle to the other. Imagine a point halfway between the two intersections—can you see how rotating about takes angle 3 to angle 5? Using what we know about vertical angles, adjacent angles, and alternate interior angles, we can find the measures of any of the eight angles created by a transversal if we know just one of them. For example, starting with the fact that angle 1 is we use vertical angles to see that angle 3 is , then we use alternate interior angles to see that angle 5 is , then we use the fact that angle 5 is supplementary to angle 8 to see that angle 8 is since . It turns out that there are only two different measures. In this example, angles 1, 3, 5, and 7 measure , and angles 2, 4, 6, and 8 measure . alternate interior angles Alternate interior angles are created when two parallel lines are crossed by another line called a transversal. Alternate interior angles are inside the parallel lines and on opposite sides of the transversal. This diagram shows two pairs of alternate interior angles. Angles and are one pair and angles and are another pair. transversal A transversal to two parallel lines is a line that cuts across them, intersecting each one. This diagram shows a transversal line intersecting parallel lines and . Use the diagram to find the measures of each angle. Explain your reasoning. Lines and are parallel, and the measure of angle is 19 degrees. Explain why the measure of angle is 19 degrees. If you get stuck, consider translating line by moving to . What is the measure of angle ? Explain. The diagram shows three lines with some marked angle measures. Find the missing angle measures marked with question marks. The two figures are scaled copies of each other. What are some ways that you can tell they are scaled copies? What is the scale factor that takes Figure 1 to Figure 2? What is the scale factor that takes Figure 2 to Figure 1? In this critical geometry lesson, you’ll learn all about parallel lines cut by a transversal. Jenn, Founder Calcworkshop®, 15+ Years Experience (Licensed & Certified Teacher) You’ll gain experience classifying line types, identifying angle relationships, and finally using that knowledge to solve problems for missing angles. Let jump in! What Are Parallel Lines? What comes to mind when you think of parallel lines? Is it the definition, which states that parallel lines are coplanar and never intersect because they are the same distance apart? Or perhaps you envision two lines that have the same slope and different y-intercepts as we learned in Algebra? Or maybe it’s just a visual image like a railroad track or a picket fence. Parallel Lines Examples What Is A Transversal? A transversal is a line that intersects two or more coplanar lines, each at a different point. What this means is that, two lines are intersected by a third line, and in so doing, creates six angle-pair relationships as demonstrated below: Interior angles: ∠3,∠4,∠5,∠6 Exterior angles:∠1,∠2,∠7,∠8 Pairs of alternate exterior angles: ∠1,∠7 ; ∠2,∠8 Pairs of alternate interior angles: ∠4,∠6 ; ∠3,∠5 Pairs of corresponding angles: ∠1,∠5 ; ∠2,∠6 ; ∠3,∠7 ; ∠4,∠8 Paris of angles on the same-side of the transversal: ∠3,∠6 ; ∠4,∠5 Transversal Line Example Parallel Lines and Transversals Postulates Parallel lines and transversals are very important to the study of geometry because they enable us to define congruent angle pair relationships. How? Well, when two parallel lines are cut by a transversal (i.e., get crossed by a third line), then not only do we notice the vertical angles and linear pairs that are subsequently formed, but the following angle pair relationships are created as well: Corresponding Angles are congruent Alternate Exterior Angles are congruent Alternate Interior Angles are congruent Same Side Interior Angles (Consecutive Interior Angles) sum to 180 degrees And knowing how to identify these angle pair relationships is crucial for proving two lines are parallel, as Study.Com accurately states. In the video below, you’ll discover that if two lines are parallel and are cut by a transversal, then all pairs of corresponding angles are congruent (i.e., same measure), all pairs of alternate exterior angles are congruent, all pairs of alternate interior angles are congruent, and same side interior angles are supplementary! Transversal Angles Corresponding Angles ∠1 is congruent to ∠5 ∠2 is congruent to ∠6 ∠3 is congruent to ∠7 ∠4 is congruent to ∠8 Corresponding Angles Alternate Exterior Angles ∠1 is congruent to ∠8 ∠2 is congruent to ∠7 Alternate Exterior Angles Alternate Interior Angles ∠3 is congruent to ∠5 ∠4 is congruent to ∠6 Alternate Interior Angles Same Side Interior Angles ∠3 and ∠6 are supplementary ∠4 and ∠5 are supplementary Same Side Interior Angles Wow! In the following video, you’ll learn all about classifying lines as parallel, intersecting, or skew. Then you’ll learn how to identify transversal lines and angle pair relationships. Next, you’ll use your knowledge of parallel lines to determine the measure of angles. And lastly, you’ll write two-column proofs given parallel lines. Parallel Lines Cut By A Transversal – Lesson & Examples (Video) 1 hr 10 min What are parallel, intersecting, and skew lines? (Examples #1-8) 00:09:34 – Overview of transversal and angle pair relationships (Examples #9-14) 00:20:52 – Theorems for perpendicular and parallel lines 00:28:47 – Find the measure of each angle given two parallel lines cut by a transversal (Examples #15-18) 00:46:05 – Find the measure of each angle (Example #19) Exclusive Content for Member’s Only 00:49:49 – From the given information can you prove the two lines are parallel? (Example #20) 00:55:01 – Write a two-column proof given parallel lines (Examples #21-23) Practice Problems with Step-by-Step Solutions Chapter Tests with Video Solutions Get access to all the courses and over 150 HD videos with your subscription Monthly, Half-Yearly, and Yearly Plans Available Get My Subscription Now Not yet ready to subscribe? Take Calcworkshop for a spin with our FREE limits course How can you prove two lines are actually parallel? As with all things in geometry, wiser, older geometricians have trod this ground before you and have shown the way. By using a transversal, we create eight angles which will help us. What Makes Lines Parallel? Two lines are parallel if they never meet and are always the same distance apart. Both lines must be coplanar (in the same plane). To use geometric shorthand, we write the symbol for parallel lines as two tiny parallel lines, like this: ∥. For example, to say line JI is parallel to line NX, we write: JI ∥ NX What are Parallel Lines in Real Life? If you have ever stood on unused railroad tracks and wondered why they seem to meet at a point far away, you have experienced parallel lines (and perspective!). If the two rails met, the train could not move forward. Other parallel lines are all around you: Street markings Crosswalks Bookshelves Notebook paper Parallel Lines Cut By A Transversal A line cutting across another line is a transversal. When cutting across parallel lines, the transversal creates eight angles. Create a transversal using any existing pair of parallel lines, by using a straightedge to draw a transversal across the two lines, like this: Proving Lines are Parallel Those eight angles can be sorted out into pairs. Let's label the angles, using letters we have not used already: Angles In Parallel Lines These eight angles in parallel lines are: Corresponding angles Alternate interior angles Alternate exterior angles Supplementary angles Every one of these has a postulate or theorem that can be used to prove the two lines MA and ZE are parallel. Let's go over each of them. Corresponding Angles The Corresponding Angles Postulate states that parallel lines cut by a transversal yield congruent corresponding angles. We want the converse of that, or the same idea the other way around: If a transversal cuts across two lines to form two congruent, corresponding angles, then the two lines are parallel. To know if we have two corresponding angles that are congruent, we need to know what corresponding angles are. In our drawing, transversal OH sliced through lines MA and ZE, leaving behind eight angles. Each slicing created an intersection. If one angle at one intersection is the same as another angle in the same position in the other intersection, then the two lines must be parallel. Two angles are corresponding if they are in matching positions in both intersections. In our drawing, the corresponding angles are: ∠B and ∠G ∠C and ∠J ∠F and ∠L ∠D and ∠K If you check only a single pair of corresponding angles and they are equal, then the two lines are parallel. Alternate Angles Alternate angles as a group subdivide into alternate interior angles and alternate exterior angles. Exterior angles lie outside the open space between the two lines suspected to be parallel. Interior angles lie within that open space between the two questioned lines. In our drawing, ∠B, ∠C, ∠K and ∠L are exterior angles. Can you identify the four interior angles? Did you say ∠D, ∠F, ∠G and ∠J? Alternate angles appear on either side of the transversal. They cannot by definition be on the same side of the transversal. In our drawing, ∠B is an alternate exterior angle with ∠L. ∠D is an alternate interior angle with ∠J. Can you find another pair of alternate exterior angles and another pair of alternate interior angles? Here are both pairs of alternate exterior angles: ∠B and ∠L ∠C and ∠K Here are both pairs of alternate interior angles: ∠D and ∠J ∠F and ∠G Alternate Exterior Angles If just one of our two pairs of alternate exterior angles are equal, then the two lines are parallel, because of the Alternate Exterior Angle Converse Theorem, which says: If two lines are cut by a transversal and the alternate exterior angles are equal, then the two lines are parallel. Angles can be equal or congruent; you can replace the word "equal" in both theorems with "congruent" without affecting the theorem. So if ∠B and ∠L are equal (or congruent), the lines are parallel. You could also only check ∠C and ∠K; if they are congruent, the lines are parallel. You need only check one pair! Alternate Interior Angles Just like the exterior angles, the four interior angles have a theorem and converse of the theorem. We are interested in the Alternate Interior Angle Converse Theorem: If two lines are cut by a transversal and the alternate interior angles are equal (or congruent), then the two lines are parallel. So, in our drawing, if ∠D is congruent to ∠J, lines MA and ZE are parallel. Or, if ∠F is equal to ∠G, the lines are parallel. Again, you need only check one pair of alternate interior angles! Supplementary Angles Supplementary angles add to 180°. Supplementary angles create straight lines, so when the transversal cuts across a line, it leaves four supplementary angles. When a transversal cuts across lines suspected of being parallel, you might think it only creates eight supplementary angles, because you doubled the number of lines. Not true! It creates more than eight! In our main drawing, can you find all 12 supplementary angles? Around the top intersection: ∠B and ∠C ∠C and ∠F ∠F and ∠D ∠D and ∠B Around the bottom intersection: ∠G and ∠J ∠J and ∠L ∠L and ∠K ∠K and ∠G Those should have been obvious, but did you catch these four other supplementary angles? ∠B and ∠K ∠L and ∠C ∠F and ∠J ∠D and ∠G These four pairs are supplementary because the transversal creates identical intersections for both lines (only if the lines are parallel). The last two supplementary angles are interior angle pairs, called consecutive interior angles. Consecutive Interior Angle Converse Theorem If two lines are cut by a transversal and the consecutive interior angles are supplementary, then the two lines are parallel. As you may suspect, if a converse Theorem exists for consecutive interior angles, it must also exist for consecutive exterior angles. Consecutive Exterior Angle Converse Theorem If two lines are cut by a transversal and the consecutive exterior angles are supplementary, then the two lines are parallel. Consecutive exterior angles have to be on the same side of the transversal, and on the outside of the parallel lines. So, in our drawing, only these consecutive exterior angles are supplementary: ∠B and ∠K ∠L and ∠C Keep in mind you do not need to check every one of these 12 supplementary angles. Just checking any one of them proves the two lines are parallel! Lesson Summary After careful study, you have now learned how to identify and know parallel lines, find examples of them in real life, construct a transversal, and state the several kinds of angles created when a transversal crosses parallel lines. Those angles are corresponding angles, alternate interior angles, alternate exterior angles, and supplementary angles. Using those angles, you have learned many ways to prove that two lines are parallel. Next Lesson: How to Construct Parallel Lines